

# Energy-momentum balance in a particle domain wall perforating collision

D. V. Gal'tsov<sup>1</sup>, E. Yu. Melkumova<sup>1</sup> and P. Spirin<sup>1,2\*</sup>

<sup>1</sup> *Department of Theoretical Physics, Faculty of Physics,  
Moscow State University, 119899, Moscow, Russia;*

<sup>2</sup> *Institute of Theoretical and Computational Physics,  
Department of Physics, University of Crete, 71003, Heraklion, Greece.*

We investigate the energy-momentum balance in the perforating collision of a point particle with an infinitely thin planar domain wall within the linearized gravity in arbitrary dimensions. Since the metric of the wall increases with distance, the wall and the particle are never free, and their energy-momentum balance involves not only the instantaneous kinetic momenta, but also the nonlocal contribution of gravitational stresses. However, careful analysis shows that the stresses can be unambiguously divided between the colliding objects leading to definition of the gravitationally dressed momenta. These take gravity into account in the same way as the potential energy does in the nonrelativistic theory, but our treatment is fully relativistic. Another unusual feature of our problem is the nonvanishing flux of the total energy-momentum tensor through the lateral surface of the world tube. In this case the zero divergence of the energy-momentum tensor does not imply conservation of the total momentum defined as the integral over the spacelike section of the tube. But one can still define the conservation law infinitesimally, passing to time derivatives of the momenta. Using this definition we establish the momentum balance in terms of the dressed particle and wall momenta.

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## 1. INTRODUCTION

In the standard theory of particle collisions, both classical and quantum, one assumes the existence of asymptotic states in which the particles can be regarded as noninteracting. This gives rise to the energy-momentum conservation playing a crucial role in the understanding of such processes. For this picture to be valid, the interaction force between the colliding objects must fall down sufficiently fast with the distance. Meanwhile, in various physically interesting situations this is not so, the notable example being interaction between quarks.

To explore the possibility of the energy-momentum definition in the absence of asymptotically free states we consider here a collision of the gravitationally interacting infinitely thin domain wall and point particle. Such a problem is of interest for applications in the standard [1–6] and the Randall-Sundrum-type [7–10] cosmology, string theory [11], in studying brane -black hole composites [12, 13], black hole escape from branes [14–16], and in other situations. Gravitational force exerted upon the particle by the plane domain wall does not fall with distance [17, 18], so the particle cannot be considered free at any moment. If the domain wall is viewed as a fixed source of gravity, the particle moves along the geodesic in the space-time generated by the domain wall, and the notion of the gravitational potential energy can be introduced. But if one wants to treat both objects on equal footing, the interaction potential cannot be introduced.

The domain wall -particle scattering problem, however, is well posed within the linearized gravity, where it can be formulated in close parallel to the case of two gravitating particles. Moreover, while the head-on collision of particles is a singular problem even in the linearized gravity, our process is still tractable, since the gravitational force acting upon the particle remains finite when it comes into contact with the wall. Recently we have shown [19, 20] that the perforation of the domain wall by the particle can be well described in linearized gravity in terms of distributions. A novel feature of this situation is due to the existence of the internal dynamics of the domain wall which gets excited after the perforation in the form of the spherical branon wave.

Here we would like to show that the problem of the energy-momentum conservation in the domain wall - particle interaction is also tractable going beyond the linear theory up to the second order in the gravitational constant. This is needed in order to introduce the effective gravitational stress tensor which has to be taken into account in establishing the energy-momentum balance. Such a stress tensor obtained by expanding the Einstein tensor up to the second order in metric deviations is nonlocal. But, as we will show, careful analysis allows one to unambiguously split it between the domain wall and the particle leading to a definition of gravitationally dressed colliding objects. This dressing resembles introduction of the potential energy in the nonrelativistic theory, but an essential difference is that now the treatment is fully relativistic and both objects are considered on equal footing. Gravitational dressing does not mean taking into account a proper gravitational field of each

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\* galtsov@phys.msu.ru, elenamelk@physics.msu.ru, pspirin@physics.uoc.gr.

object, but rather accounting for the gravitational field of the partner. Therefore, our dressing must not be confused with the self-energy problem.

## 2. THE SETUP

Our system consists of a point particle moving along the world line  $x^M = z^M(\tau)$  and an infinitely thin domain wall filling the world volume  $\mathcal{V}_{D-1}$  given by the embedding equations  $x^M = X^M(\sigma^\mu)$  in  $D$ -dimensional space-time with the metric  $g_{MN}$ ,  $M = 0, \dots, D-1$ ,  $\mu = 0, \dots, D-2$  of the signature  $(+, -, \dots, -)$ . The action can be written as

$$S = S_p + S_{dw} + S_{grav}, \quad (2.1)$$

where  $S_p(z^M, e)$  is the particle action in the Polyakov form

$$S_p = -\frac{1}{2} \int \left( e g_{MN} \dot{z}^M \dot{z}^N + \frac{m^2}{e} \right) d\tau, \quad (2.2)$$

[ $e(\tau)$  is the einbein on the particle world line],  $S_{dw}(X^M, \gamma_{\mu\nu})$  is the domain wall geometrical action

$$S_{dw} = -\frac{\mu}{2} \int \left[ X_\mu^M X_\nu^N g_{MN} \gamma^{\mu\nu} - (D-3) \right] \sqrt{|\gamma|} d^{D-1}\sigma, \quad (2.3)$$

where  $X_\mu^M = \partial X^M / \partial \sigma^\mu$  are the tangent vectors and  $\gamma^{\mu\nu}$  is the inverse metric on the domain wall world volume  $\mathcal{V}_{D-1}$ ,  $\gamma = \det \gamma_{\mu\nu}$ , and  $S_{grav}(g_M)$  is the Einstein-Hilbert action reads

$$S_{grav} = -\frac{1}{\kappa^2} \int R_D \sqrt{|g|} d^D x, \quad \kappa^2 \equiv 16\pi G_D. \quad (2.4)$$

Variation of (2.3) with respect to  $X^M$  and  $\gamma^{\mu\nu}$  gives the brane equation of motion in the covariant form

$$\partial_\mu \left( X_\nu^N g_{MN} \gamma^{\mu\nu} \sqrt{|\gamma|} \right) = \frac{1}{2} g_{NP,M} X_\mu^N X_\nu^P \gamma^{\mu\nu} \sqrt{|\gamma|}, \quad (2.5)$$

and the constraint equation

$$\left( X_\mu^M X_\nu^N - \frac{1}{2} \gamma_{\mu\nu} \gamma^{\lambda\tau} X_\lambda^M X_\tau^N \right) g_{MN} + \frac{D-3}{2} \gamma_{\mu\nu} = 0, \quad (2.6)$$

whose solution defines  $\gamma_{\mu\nu}$  as the induced metric on  $\mathcal{V}_{D-1}$ :

$$\gamma_{\mu\nu} = X_\mu^M X_\nu^N g_{MN} \big|_{x=X}.$$

Similarly, variation of (2.2) with respect to  $z^M(\tau)$  and  $e(\tau)$  gives the geodesic equation in arbitrary parametrization

$$\frac{d}{d\tau} (e \dot{z}^N g_{MN}) = \frac{e}{2} g_{NP,M} \dot{z}^N \dot{z}^P, \quad (2.7)$$

and the constraint

$$e^2 g_{MN} \dot{z}^M \dot{z}^N = m^2. \quad (2.8)$$

We prefer to keep the Lagrange multipliers explicitly to facilitate formulation of the perturbation theory.

Finally, variation of the Einstein-Hilbert action (2.4) over  $g_{MN}$  leads to the Einstein equations

$$G^{MN} = \frac{1}{2} \kappa^2 \left[ T^{MN} + \bar{T}^{MN} \right], \quad (2.9)$$

with the source terms due to the domain wall

$$T^{MN} = \mu \int X_\mu^M X_\nu^N \gamma^{\mu\nu} \frac{\delta^D(x - X(\sigma))}{\sqrt{|g|}} \sqrt{|\gamma|} d^{D-1}\sigma, \quad (2.10)$$

and the particle (the corresponding quantities here and below will be labeled by bar):

$$\bar{T}^{MN} = e \int \frac{\dot{z}^M \dot{z}^N \delta^D(x - z(\tau))}{\sqrt{|g|}} d\tau. \quad (2.11)$$

Einstein equations with the source term (2.10) have some exact nonsingular solutions [17, 18, 21–23], while no such solutions are possible for the point particle source. Actually, reasonable exact solutions exist for branes embedded into space-time with codimensions one and two, but not higher. In any case we need here time-dependent solutions describing the collision, which can only be constructed perturbatively. We work in linearized gravity assuming smallness of deviation of the space-time metric from Minkowskian:

$$g_{MN} = \eta_{MN} + \varkappa H_{MN}, \quad (2.12)$$

but we keep the full Einstein action to be able to extract the gravitational stress tensor as the second-order expansion term of the Einstein tensor:

$$G^{MN} = -\frac{\varkappa}{2} \square \left( H^{MN} - \frac{1}{2} \eta^{MN} H \right) - \frac{\varkappa^2}{2} S^{MN} + O(H^3), \quad (2.13)$$

where  $H = H_M^M$ ,  $\square = \eta^{MN} \partial_M \partial_N$ , and  $S^{MN}$  stands for the quadratic terms in  $H_{MN}$ :

$$\begin{aligned} S^{MN} = & 2 H^{MP,Q} H^N_{[Q,P]} + H_{PQ} \left( H^{MP,NQ} + H^{NP,MQ} - H^{PQ,MN} - H^{MN,PQ} \right) - 2 H_P^{(M} \square H^{N)P} - \\ & - \frac{1}{2} H^{PQ,M} H_{PQ}{}^{,N} + \frac{1}{2} H^{MN} \square H + \frac{1}{2} \eta^{MN} \left( 2 H^{PQ} \square H_{PQ} - H_{PQ,L} H^{PL,Q} + \frac{3}{2} H_{PQ,L} H^{PQ,L} \right). \end{aligned} \quad (2.14)$$

In this definition there is the following subtlety. The metric deviation  $H_{MN}$  is defined initially as generally covariant quantity with *lower* indices and then identified with the Minkowskian tensor whose indices are raised with the inverse Minkowski metric. The quadratic tensor  $S^{MN}$ , which is also further regarded as the Minkowskian tensor, is obtained expanding the Einstein tensor with *upper* indices, all internal contractions of metric deviations being performed with Minkowski metric.

The full set of variables in our problem consists of  $z^M(\tau)$ ,  $e(\tau)$ ,  $X^\mu(\sigma)$ ,  $\gamma_{\mu\nu}$ , and  $H_{MN}(x)$ . To treat the problem perturbatively we expand all of them in powers of  $\varkappa$  and derive the system of iterative equations. The  $D$ -dimensional Cartesian coordinates of the embedding space-time are split as  $x^M = (x^\mu, z)$ ,  $x^\mu = (t, \mathbf{r})$ , and the particle is assumed to move along  $z$ , i.e. normally to the domain wall. In the zeroth order the particle is assumed to move with the constant velocity

$$u^M = \gamma(1, 0, \dots, 0, v), \quad \text{where } \gamma = 1/\sqrt{1-v^2},$$

so the world line and the einbein are

$$z^M(\tau) = u^M \tau, \quad e = \text{const} = m,$$

corresponding to the parametrization in terms of the proper time. The wall in the zeroth order is assumed to be plane, unexcited and being at rest at  $z = 0$  in the chosen Lorentz frame:

$$X^M = \Sigma_\mu^M \sigma^\mu,$$

where  $\Sigma_\mu^M$  are  $(D-1)$  constant Minkowski vectors normalized as

$$\Sigma_\mu^M \Sigma_\nu^N \eta_{MN} = \eta_{\mu\nu}. \quad (2.15)$$

Obviously, this is a solution to the Eq. (2.5) for  $\varkappa = 0$ , and the corresponding induced metric is the four-dimensional Minkowski metric  $\gamma_{\mu\nu} = \eta_{\mu\nu}$ . Thus it is convenient to fix  $\Sigma_\mu^M = \delta_\mu^M$  without loss of generality. The moment of perforation of the wall by the particle that occurs at  $z = 0$  is  $t = \tau = 0$ .

The metric deviation must be further expanded in  $\varkappa$ :

$$H^{MN} = h^{MN} + \bar{h}^{MN} + \delta H^{MN}, \quad (2.16)$$

where the first-order term is split into the sum of contributions of the wall  $h^{MN}$  and of the particle  $\bar{h}^{MN}$ . These obey the linear equations

$$\square h^{MN} = -\varkappa \left( T^{MN} - \frac{1}{D-2} \eta^{MN} T_P^P \right), \quad \square \bar{h}^{MN} = -\varkappa \left( \bar{T}^{MN} - \frac{1}{D-2} \eta^{MN} \bar{T}_P^P \right), \quad (2.17)$$

where the sources must be constructed in terms of the above zeroth order quantities and the Fock-deDonder gauge  $\partial_N h^{MN} - 1/2 \partial^M h = 0$  is assumed. The next order metric deviation  $\delta H^{MN}$  does not split anymore on separate contributions and obeys (in the same gauge) the d'Alembert equation

$$\square \left( \delta H^{MN} - \frac{1}{2} \eta^{MN} \delta H \right) = -\kappa \tau^{MN}, \quad (2.18)$$

with the source

$$\tau^{MN} = \delta T^{MN} + \delta \bar{T}^{MN} + S^{MN}(h, \bar{h}), \quad (2.19)$$

where  $\delta T^{MN}, \delta \bar{T}^{MN}$  are the perturbations of the wall and particle stress tensors, while  $S^{MN}(h, \bar{h})$  stands for the quadratic form  $S^{MN}$  in which  $H^{MN}$  must be taken as the sum  $H^{MN} = h^{MN} + \bar{h}^{MN}$  keeping only the crossed terms in  $h^{MN}, \bar{h}^{MN}$ . The quantity  $S^{MN}(h, \bar{h})$  is regarded as the gravitational stress tensor whose presence is needed to ensure the fulfillment of the conservation equation up to the first order in  $\kappa$ :

$$\partial_N \tau^{MN} = 0. \quad (2.20)$$

Generically the gravitational stress tensor is a nonlocal quantity, but, as we will show below, within the perturbation theory it can still be split into two contributions which may be attributed to the wall and the particle separately. This is how the idea of gravitational dressing is implemented.

The domain of validity of our perturbation theory is somewhat subtle and worthwhile being discussed in detail. Gravity force exerted by the wall upon the particle is repulsive and we consider the case when the initial velocity of the particle is large enough to reach the wall and to perforate it. After the perforation the particle gets accelerated by the wall's gravitational repulsion and goes away. Since the metric deviations caused by the wall in the linearized gravity are growing with  $z$ , one can treat the collision perturbatively only in some vicinity of the perforation moment. From the particle energy  $\mathcal{E} = m\gamma$ , the wall's tension  $\mu$  (of dimensionality  $\text{length}^{-(D-1)}$ ) and the gravitational coupling constant  $\kappa^2$ , having in  $D$  dimensions the dimensionality  $\text{length}^{D-2}$ , one can form two length parameters (in the units  $c = 1$ ):

$$l \simeq [\kappa^2 \mu]^{-1}, \quad r_S \simeq (\kappa^2 \mathcal{E})^{\frac{1}{D-3}}, \quad (2.21)$$

the first of which corresponds to the curvature radius of the bulk generated by the wall, while the second is the gravitational radius of the energy  $\mathcal{E}$ . The wall's gravity is small at the distances from the wall  $z < l$ , while gravity of the particle is small for  $z^2 + r^2 > r_S^2$ . If  $r_S \ll l$ , these conditions intersect within some matching zone. But it turns out [19] that, assuming the linearized gravity to be true gravity theory for point particles *elsewhere* one is still able to treat the collision up to  $z = 0$  (the perforation point) consistently in terms of distributions (i.e. in the formal limit  $r_S = 0$ ). Here we will show that this treatment is consistent with the energy-momentum conservation in the perforation process in the linear order in  $\kappa$ , thus giving further evidence for validity of our approach. Note that this is quite different from the more singular case of the head-on collision of two gravitating point particles that cannot be treated within the linearized gravity.

### 3. FIRST-ORDER PERTURBATIONS

For reader's convenience we briefly reproduce here the results obtained in [19]. The full metric deviation in the first order is the sum of  $\bar{h}_{MN}$  generated by the unperturbed particle motion and  $h_{MN}$  representing gravity of the unperturbed wall at rest. The first reads explicitly for  $D > 3$ :

$$\bar{h}_{MN}(x) = -\frac{\kappa m \Gamma(\frac{D-3}{2})}{4\pi^{\frac{D-1}{2}}} \left( u_M u_N - \frac{1}{D-2} \eta_{MN} \right) \frac{1}{[\gamma^2(z - vt)^2 + r^2]^{\frac{D-3}{2}}}, \quad (3.1)$$

where  $r = \sqrt{\delta_{ij} \sigma^i \sigma^j}$  is the radial distance on the wall from the perforation point. This is just the Lorentz-contracted  $D$ -dimensional Newton field of the uniformly moving particle. In what follows we will also need the corresponding Fourier transform

$$\bar{h}_{MN}(q) = \int e^{iqx} \bar{h}_{MN}(x) d^D x = \frac{2\pi \kappa m \delta(qu)}{q^2 + i\epsilon q^0} \left( u_M u_N - \frac{1}{D-2} \eta_{MN} \right). \quad (3.2)$$

The metric deviations due to the wall grow linearly with the distance

$$h_{MN} = \frac{\kappa \mu}{2} \left( \Xi_{MN} - \frac{D-1}{D-2} \eta_{MN} \right) |z| = \frac{2k|z|}{\kappa} \text{diag}(-1, 1, \dots, 1, D-1), \quad (3.3)$$

where

$$\Xi_{MN} = \Sigma_M^\mu \Sigma_N^\nu \eta_{\mu\nu}, \quad k \equiv \frac{\varkappa^2 \mu}{4(D-2)}, \quad (3.4)$$

and the corresponding Fourier transform reads

$$h_{MN}(q) = \frac{(2\pi)^{D-1} \varkappa \mu}{q^2} \left( \Xi_{MN} - \frac{D-1}{D-2} \eta_{MN} \right). \quad (3.5)$$

The first order correction to the particle motion  $\delta z^M$  in the field of the wall (3.3) depends on the choice of the parameter on the world line. Specifying it so that the deviation of the einbein is zero,

$$\delta e = -\frac{m}{2} \left( \varkappa h_{MN} u^M u^N + 2 \eta_{MN} u^M \delta z^N \right) = 0, \quad (3.6)$$

we obtain from the geodesic equation (2.7)

$$\delta \ddot{z}^0 = 2kv\gamma^2 \operatorname{sgn}(\tau), \quad \delta \ddot{z} \equiv \ddot{z}^{D-1} = k(D\gamma^2 v^2 + 1) \operatorname{sgn}(\tau), \quad (3.7)$$

observing that the force is repulsive as expected. Integrating (3.7) twice with initial conditions  $\delta z^M(0) = 0$ ,  $\delta \dot{z}^M(0) = 0$ , one has

$$\delta z^0 = kv\tau^2 \gamma^2 \operatorname{sgn}(\tau), \quad \delta z = \frac{1}{2} k\tau^2 (D\gamma^2 v^2 + 1) \operatorname{sgn}(\tau). \quad (3.8)$$

Substituting (3.8) back into (3.6) one can check that the gauge condition  $\delta e = 0$  holds indeed.

In order to find perturbations of the domain wall embedding functions  $\delta X^M$  due to gravitational interaction with the particle one has to derive the linearized perturbation of the Nambu-Goto equation specifying the world volume metric as an induced metric

$$\delta \gamma_{\mu\nu} = 2 \delta_{(\mu}^M \delta X_{\nu)}^N \eta_{MN} + \varkappa \bar{h}_{MN} \Sigma_\mu^M \Sigma_\nu^N, \quad (3.9)$$

where brackets denote symmetrization over indices with the factor 1/2. Then linearizing the rest of Eq. (2.5), after some rearrangements one obtains the following equation for deformation of the wall:

$$\Pi_{MN} \square_{D-1} \delta X^N = \Pi_{MN} J^N, \quad \Pi^{MN} \equiv \eta^{MN} - \Sigma_\mu^M \Sigma_\nu^N \eta^{\mu\nu}, \quad (3.10)$$

where  $\square_{D-1} \equiv \partial_\mu \partial^\mu$  and  $\Pi^{MN}$  is the projector onto the (one-dimensional) subspace orthogonal to  $\mathcal{V}_{D-1}$ . The source term in (3.10) reads:

$$J^N = \varkappa \Sigma_P^\mu \Sigma_Q^\nu \eta_{\mu\nu} \left( \frac{1}{2} \bar{h}^{PQ,N} - \bar{h}^{NP,Q} \right)_{z=0}. \quad (3.11)$$

Using the aligned coordinates on the brane  $\sigma^\mu = (t, \mathbf{r})$ , we will have  $\delta_\mu^M = \Sigma_\mu^M$ , so the projector  $\Pi^{MN}$  reduces the system (3.10) to a single equation for the  $M = z$  component. Generically, the transverse coordinates of the branes can be viewed as Nambu-Goldstone bosons (branos) that appear as a result of spontaneous breaking of the translational symmetry [24]. These are coupled to gravity and matter on the brane in the brane-world models via the induced metric [25]. In our case of the codimension one, there is only one such branon. The remaining components of the perturbation  $\delta X^M$  can be removed by suitable transformation of the coordinates on the world volume, so the equality  $\delta X^\mu = 0$  is nothing but the choice of gauge. Note that in this gauge the perturbation of the induced metric  $\delta \gamma_{\mu\nu}$  does not vanish, as it was for the perturbation of the particle einbein  $e$ .

Denoting the physical component as  $\delta X^z \equiv \Phi(\sigma^\mu)$  we obtain the branon  $(D-1)$ -dimensional wave equation:

$$\square_{D-1} \Phi(\sigma^\mu) = J(\sigma^\mu), \quad (3.12)$$

with the source term  $J \equiv J^z$ . Substituting (3.1) into the Eq. (3.11) we obtain

$$J(\sigma) = \varkappa \left( \frac{1}{2} \eta_{\mu\nu} \bar{h}^{\mu\nu,z} - \bar{h}^{z0,0} \right)_{z=0} = -\frac{\lambda v t}{[\gamma^2 v^2 t^2 + r^2]^{\frac{D-1}{2}}}, \quad (3.13)$$

with

$$\lambda = \frac{\varkappa^2 m \gamma^2 \Gamma(\frac{D-1}{2})}{4\pi^{\frac{D-1}{2}}} \left( \gamma^2 v^2 + \frac{1}{D-2} \right). \quad (3.14)$$

The retarded solution to Eq. (3.12) consists of two parts  $\Phi = \Phi_a + \Phi_b$ , where the first is antisymmetric in time and represents an eventual deformation of the wall correlated with the particle motion. The second part is the spherical branon wave starting at the moment of perforation and propagating to infinity with the velocity of light. This wave is not the solution of the homogeneous branon equation, but it has a jump at  $t = 0$  ensuring continuity of the full solutions. The explicit expressions of both parts were presented in [19]; they depend on the dimension of the space-time. Here we will not need their explicit form, so we give only their integral representations suitable for later use:

$$\Phi_a \equiv -\Lambda \operatorname{sgn}(t) I_a, \quad \Phi_b \equiv 2 \Lambda \theta(t) I_b, \quad \Lambda \equiv \frac{\sqrt{\pi} \lambda}{2^{\frac{D-2}{2}} \gamma^3 \Gamma\left(\frac{D-1}{2}\right)}, \quad (3.15)$$

$$I_a(t, r) = \frac{1}{r^{\frac{D-4}{2}}} \int_0^\infty dk J_{\frac{D-4}{2}}(kr) k^{\frac{D-6}{2}} e^{-k\gamma v|t|}, \quad (3.16)$$

$$I_b(t, r) = \frac{1}{r^{\frac{D-4}{2}}} \int_0^\infty dk J_{\frac{D-4}{2}}(kr) k^{\frac{D-6}{2}} \cos kt, \quad (3.17)$$

where  $J_\nu(z)$  is a Bessel function of the first kind.

#### 4. CONSERVATION OF THE ENERGY-MOMENTUM

In the first order in  $\varkappa$  the total energy-momentum tensor consists of three contributions (2.18) and satisfies the conservation equation (2.19). To convert the latter into the energy-momentum balance equation one has to integrate over the world tube  $\Omega$ :

$$0 = \int_\Omega \partial_N \tau^{MN} = \int_{\partial\Omega} \tau^{MN} d\Sigma_N, \quad (4.1)$$

bounded by the closed hypersurface

$$\partial\Omega = \Sigma_{t_0} \cup \Sigma_{t_f} \cup \Sigma_\infty, \quad (4.2)$$

consisting of two spacelike hypersurfaces associated with the moments of time  $t_0, t_f$  (usually chosen orthogonal to the time axis), and the closing lateral hypersurface  $\Sigma_\infty$  at spatial infinity. To get the usual energy-momentum conservation equation two conditions should hold: i) finiteness of the integral of  $\tau^{MN}$  over  $\Sigma_t$  which is interpreted as the  $D$ -momentum vector,

$$P_{\text{tot}}^M(t) = \int_{\Sigma_t} \tau^{MN} d\Sigma_N = \int \tau^{M0} dz d^{D-2}\mathbf{r}, \quad (4.3)$$

and ii) vanishing of the lateral flux  $\tau^{MN}$  through the timelike hypersurface  $\Sigma_\infty$ . This is usually guaranteed by the sufficient falloff of the integrand at infinity. In the case of the domain wall both conditions are not satisfied. First, the wall is considered an infinite and having finite mass density, so the total energy in the zero order in  $\varkappa$  diverges. We will see shortly that the corresponding contribution diverges also in the linear in  $\varkappa$  order. Secondly, the lateral flux for the wall is nonzero since the integrand does not fall fast enough at spatial infinity. So in our case the momentum equation reads

$$P_{\text{tot}}^M(t_f) - P_{\text{tot}}^M(t_0) = - \int_{\Sigma_\infty} \tau^{MN} d\Sigma_N. \quad (4.4)$$

According to the split of the total energy-momentum tensor (2.19) we can write

$$P_{\text{tot}}^M(t) = \delta \bar{P}^M(t) + \delta P^M(t) + S^M(t), \quad (4.5)$$

where

$$\delta \bar{P}^M(t) = \int \delta \bar{T}^{M0} dz d^{D-2}\mathbf{r}, \quad (4.6)$$

$$\delta P^M(t) = \int \delta T^{M0} dz d^{D-2}\mathbf{r} \quad (4.7)$$

are the first-order kinetic momenta carried by the particle <sup>1</sup> and the wall, while

$$S^M(t) = \int \delta S^{M0} dz d^{D-2}\mathbf{r} \quad (4.8)$$

is the momentum carried by their gravitational field. The lateral flux at the right-hand side of Eq. (4.4) can also be split into three similar contributions. The boundary hypersurface  $\Sigma_\infty$  in the  $(D-1)$ -dimensional space consists of three components

$$\Sigma_\infty = T \times (B_- \cup B_+ \cup D_R), \quad (4.9)$$

where  $T$  is the time real axis, and  $B_\pm, D_R$  are the  $(D-2)$ -dimensional surfaces:  $B_\pm = \{\text{all } \mathbf{r}, z \rightarrow \pm\infty\}$  (with an associated measure  $d^{D-2}\mathbf{r}$ ) and  $D_R = \{\text{all } z, R = |\mathbf{r}| \rightarrow \infty\}$  (with the measure  $R^{D-3}d^{D-3}\Omega$ ). Actually, all the fluxes through  $B_\pm$  vanish, as well as the fluxes of  $\delta\bar{T}^{Mr}, \delta S^{Mr}$  through  $D_R$ , but not the flux  $\delta T^{Mr}$  representing the contribution of the wall. We are therefore left with

$$P_{\text{tot}}^M(t_f) - P_{\text{tot}}^M(t_0) = - \lim_{R \rightarrow \infty} \left( \int_{t_0}^{t_f} dt \int_{-\infty}^{\infty} dz \int_{S_R^{D-3}} \delta T^{Mr} R^{D-3} d^{D-3}\Omega \right). \quad (4.10)$$

Thus the difference between the momenta defined in a standard way as the integrals over spacelike hypersurfaces is related to some integral over the corresponding time interval. Another unpleasant feature is that the integrals (4.7) are divergent for an infinite wall. To cure both of these drawbacks one could introduce the cutoff volume for the wall, but this make the analysis more complex. Instead we pass to time derivatives of the momenta which are all finite. In other words we check the momentum conservation between the infinitely close moments of time. Then the Eq. (4.10) gives

$$\frac{d}{dt} (\delta\bar{P}^M(t) + \delta P^M(t) + S^M(t)) = - \lim_{R \rightarrow \infty} \left( \int_{-\infty}^{\infty} dz \int_{S_R^{D-3}} \delta T^{Mr} R^{D-3} d^{D-3}\Omega \right) \equiv f^M, \quad (4.11)$$

where the integral at the right-hand side will be called the lateral momentum flux. This term looks like an external force acting upon the system, but in fact it is due to an additional loss of the wall momentum. In principle it could be absorbed by the redefinition of the wall momentum at the left-hand side, but we prefer to keep the usual definition (4.7).

## 5. COMPUTATION OF THE MOMENTA

We proceed in analyzing various contributions to the differential conservation equation (4.11). Note that in the *zero* order in  $\varkappa$  the particle and the wall kinetic momenta are simply

$$\bar{P}^M = m u^M, \quad P^M = \mu V_{\text{br}} \delta_0^M,$$

where  $V_{\text{br}}$  is the world volume introduced for normalization. These quantities are constant which can be omitted from further analysis.

### A. Kinetic momenta

The first-order particle stress tensor is obtained expanding the general expression (2.11) in  $\varkappa$ :

$$\delta\bar{T}^{MN}(x) = \frac{m}{2} \int \left[ 4\delta z^{(M} u^{N)} - u^M u^N \left( \varkappa h + 2\delta z^P \partial_P \right) \right] \delta^D(x - u\tau) d\tau, \quad (5.1)$$

where  $h$  is the trace of the first-order metric deviation due to the wall (3.3); the symmetrization over the indices  $(MN)$  as well as the antisymmetrization  $[MN]$  below is defined with  $1/2$ . The delta function indicates the localization of the integrand at the nonperturbed particle world line. (Note that our integral definition

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<sup>1</sup> As we have already noted, the particle kinetic momenta defined as the integral (4.6) does not coincide with the generalized Hamiltonian momentum  $m\dot{z}^N g_{MN}$  once gravity is taken into account. Our present definition, however, is more convenient for the further analysis.

of the kinetic momentum coincides with the Hamiltonian definition of the covariant generalized momentum  $P_M^{(h)} = \partial L / \partial \dot{z}^m$  only in the zero order in the gravitational constant.)

The first-order stress tensor of the wall is obtained substituting the first-order metric deviation (3.1) due to the particle and the first-order perturbations of the wall world volume into Eq. (2.10):

$$\delta T^{MN}(x) = \frac{\mu}{2} \int \left[ 4 \delta_\mu^{(M} \delta X_\nu^{N)} \eta^{\mu\nu} - 2 \delta_\mu^M \delta_\nu^N \left( \varkappa \bar{h}^{\mu\nu} + 2 \eta^{LR} \delta_R^{(\mu} \delta X_L^{\nu)} \right) + \delta_\mu^M \delta_\nu^N \eta^{\mu\nu} \left( \varkappa \bar{h}_\lambda^\lambda - \varkappa \bar{h} + 2 \delta X_\lambda^L \delta_L^\lambda - 2 \delta X^L \partial_L \right) \right] \delta^{D-1}(x - \sigma) \delta(z) d^{D-1}\sigma. \quad (5.2)$$

Again, the delta functions in the integrand indicate its localization on the unperturbed wall world volume.

Due to the kinematics of the collision, the *first-order* kinetic momenta also have nonzero only the 0 and  $z$  components. The particle momentum is calculated substituting the wall metric deviation  $h_{MN}(\tau)$  given by (3.3) and the particle world line deviation  $\delta z^M(\tau)$  given by (3.8) into (5.1) and integrating with the help of the delta function:

$$\delta \bar{P}^z = mk \left[ (3D - 2) \gamma v^2 + \gamma^{-1} \right] |t|, \quad \delta \bar{P}^0 = 2Dmk\gamma v |t|. \quad (5.3)$$

Now calculate the time component of the wall momentum. Substituting the deviation  $\delta X^M = \delta_z^M \Phi$  with  $\Phi$  given by (3.15) into the integrand of (4.7) we get

$$\delta T^{00} = \frac{\mu}{2} \left[ \left( -2 \varkappa \bar{h}_{00} + \varkappa \bar{h}_{zz} \right) \delta(z) - 2 \Phi \delta'(z) \right]. \quad (5.4)$$

Since  $\Phi$  is the function of the world-volume coordinates  $(t, r)$  only, the term  $\Phi \delta'(z)$  vanishes upon integration over  $z$ , so  $\delta P^0$  does not depend on  $\Phi$ . Substituting into the second quantity the particle metric deviation (3.1) one gets

$$\delta T^{00} = \frac{\Gamma\left(\frac{D-3}{2}\right)}{2\pi^{\frac{D-1}{2}}} \left( (D-2) \gamma^2 v^2 + (2D-7) \right) mk \chi \delta(z), \quad (5.5)$$

$$\chi \equiv \frac{1}{[\gamma^2(z-vt)^2 + r^2]^{\frac{D-3}{2}}}, \quad (5.6)$$

so the first-order zero component will read

$$\delta P^0 = \frac{\Gamma\left(\frac{D-3}{2}\right)}{\sqrt{\pi} \Gamma\left(\frac{D-2}{2}\right)} \left( (D-2) \gamma^2 v^2 + (2D-7) \right) mk Q, \quad (5.7)$$

where the integral of  $\chi$  over  $r$  including the volume factor

$$Q(a) = \int_0^\infty \frac{r^{D-3} dr}{(a^2 + r^2)^{\frac{D-3}{2}}}, \quad (5.8)$$

with  $a^2 = \gamma^2(z-vt)^2$ , linearly diverges at the upper limit. This is not surprising taking into account an infinite extension of the wall. To avoid a cumbersome normalization procedure, we pass from the momentum to its time derivative. This will be sufficient to define gravitational dressing of the kinetic momenta which is our main goal here. The derivative  $\dot{Q}$  is finite and the corresponding integral is easily evaluated by the substitution  $1 + (r/a)^2 = 1/y$  leading to Euler's beta function:

$$\int \frac{r^{D-3} dr}{(a^2 + r^2)^{\frac{D-1}{2}}} = \frac{1}{|a|} \frac{\sqrt{\pi} \Gamma\left(\frac{D-2}{2}\right)}{2 \Gamma\left(\frac{D-1}{2}\right)}. \quad (5.9)$$

Since the unperturbed momentum of the wall is constant (also infinite), we can interpret the resulting quantity as describing the derivative of the full momentum up to the first order simply by omitting  $\delta$ :

$$\dot{P}^0 = -\gamma v \left( (D-2) \gamma^2 v^2 + 2D-7 \right) mk \operatorname{sgn}(t). \quad (5.10)$$

The computation of the spatial component of the wall momentum is more involved. The flux  $T^{z0}$  can be simplified as follows:

$$\delta T^{0z} = \mu \Phi_{,0}(t, r) \delta(z), \quad (5.11)$$



where the wall perturbation is the sum of two terms  $\Phi = \Phi_a + \Phi_b$ , the first describing the regular deformation induced by the particle gravitational field, and the second corresponding to the shock branon wave emerging at the moment of piercing and then freely propagates outwards along the wall. Substituting (3.15) into (5.11) and taking into account that  $I_a|_{t=0} = I_b|_{t=0}$ <sup>2</sup>, one can verify the absence of terms proportional to  $\delta(t)$  in the time derivative of the total perturbation  $\Phi_{,0}$ . Thus one can write  $\delta P^z(t) = \delta P_a^z(t) + \delta P_b^z(t)$  with

$$\delta P_a^z = -\Lambda \mu \operatorname{sgn}(t) \int I_{a,0}(t, r) d^{D-2} \mathbf{r}, \quad \delta P_b^z = 2 \Lambda \mu \theta(t) \int I_{b,0}(t, r) d^{D-2} \mathbf{r}. \quad (5.12)$$

Let us start with the "regular" part  $P_a^z$ . Substituting  $I_a$  (3.16) and performing an integration over the sphere we obtain:

$$\delta P_a^z = \frac{2 \pi^{\frac{D-2}{2}} \mu \Lambda \gamma v}{\Gamma(\frac{D-2}{2})} \int J_{\frac{D-4}{2}}(kr) k^{\frac{D-4}{2}} e^{-k\gamma v|t|} r^{\frac{D-2}{2}} dk dr. \quad (5.13)$$

Using the integral

$$\int_0^\infty J_m(kr) k^m e^{-ak} dk = \frac{(2r)^m \Gamma(m + \frac{1}{2})}{\sqrt{\pi} (a^2 + r^2)^{m+1/2}}, \quad (5.14)$$

one obtains again the divergent quantity

$$\delta P_a^z = \frac{mkv}{\sqrt{\pi}} \frac{\Gamma(\frac{D-3}{2})}{\Gamma(\frac{D-2}{2})} \left( (D-2)\gamma^2 v^2 + 1 \right) Q(a), \quad (5.15)$$

now with  $a^2 = \gamma^2 v^2 t^2$ . Passing to the time derivative we use the fact that  $\delta(t) \dot{I}_a = 0$  in the distributional sense. Taking into account that there is no zero-order contribution to  $P^z$ , we can write

$$\dot{P}_a^z = -mk \left( (D-2)\gamma^2 v^2 + 1 \right) \gamma v^2 \operatorname{sgn}(t). \quad (5.16)$$

Now we present the corresponding quantities for the particle, differentiating the sum of the zero and the first-order (5.3) momenta:

$$\bar{F}^z \equiv \dot{P}^z = mk \left[ (3D-2)\gamma v^2 + \gamma^{-1} \right] \operatorname{sgn}(t), \quad \bar{F}^0 \equiv \dot{P}^0 = 2Dmk\gamma v \operatorname{sgn}(t). \quad (5.17)$$

All the momenta derivatives (5.10), (5.16), and (5.17) are constant before and after the moment of piercing  $t = 0$  when they change the sign. The sum  $\dot{P}^M + \dot{P}^M$  does not vanish for both values of  $M$ . This is not surprising since we still need to add contribution of the gravitational stresses.

## B. Branon contribution

One can check that the shock wave (branon) part of the wall's perturbation  $\Phi_b$  does not give contribution to the zero component of the momentum (the energy). However there is still the branon contribution to  $P^z$  arising after the perforation. Substituting the integral representation (3.17) for  $I_b$  into the Eq. (5.12), one obtains

$$\delta P_b^z = -2 \Lambda \mu \Omega_{D-3} \theta(t) \int k^{\frac{D-4}{2}} J_{\frac{D-4}{2}}(kr) \sin kt r^{\frac{D-2}{2}} dk dr. \quad (5.18)$$

Integration over  $k$  is performed using the integral [26]

$$\int_0^\infty k^\nu J_\nu(kr) \sin kt dk = \frac{\sqrt{\pi} (2r)^\nu (t^2 - r^2)^{-(\nu+1/2)}}{\Gamma(1/2 - \nu)} \theta(|t| - r),$$

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<sup>2</sup> This follows from (3.16, 3.17) and is explained in detail in Eq. (5.39) of [19].

to yield

$$\delta P_b^z = -\frac{2^{\frac{D}{2}} \pi^{\frac{D-1}{2}}}{\Gamma(\frac{D-2}{2}) \Gamma(-\frac{D-5}{2})} \Lambda \mu \theta(t) \int (t^2 - r^2)^{-(D-3)/2} \theta(|t| - r) r^{D-3} dr. \quad (5.19)$$

The latter expression contains  $\Gamma(-\frac{D-5}{2})$  which has a simple pole at odd  $D \geq 5$ . Thus (5.19) drastically depends upon the parity of  $D$ .

First let us consider odd  $D \geq 5$ . Applying the distributional limit [Sec. 3.5, Eq. (1) of [27]].

$$\lim_{\lambda \rightarrow -n} \frac{[x \theta(x)]^{\lambda-1}}{\Gamma(\lambda)} = \frac{d^n}{dx^n} \delta(x) \equiv \delta^{(n)}(x), \quad (5.20)$$

one gets

$$\begin{aligned} \delta P_b^z &= -\frac{2^{D/2} \pi^{\frac{D-1}{2}}}{\Gamma(\frac{D-2}{2})} \Lambda \mu \theta(t) \int \delta^{(\frac{D-5}{2})}(t^2 - r^2) r^{D-3} dr \\ &= -\frac{2^{\frac{D-2}{2}} \pi^{\frac{D-1}{2}}}{\Gamma(\frac{D-2}{2})} \Lambda \mu \theta(t) \left( \frac{\partial}{\partial t^2} \right)^{\frac{D-5}{2}} \int \delta(t - r) r^{D-4} dr, \end{aligned} \quad (5.21)$$

where the order of derivative is integer and we pass to the differentiation over  $t^2$ . Integrating trivially over  $r$  and next differentiating with respect to  $|t|^2$  according to

$$\frac{d^\lambda}{dx^\lambda} x^\rho = \frac{\Gamma(\rho+1)}{\Gamma(\rho-\lambda+1)} x^{\rho-\lambda}, \quad (5.22)$$

easily verified for integer  $\lambda$ , one obtains finally:

$$\delta P_b^z = -\frac{\kappa^2 \mu m}{2\gamma} \left( \gamma^2 v^2 + \frac{1}{D-2} \right) t \theta(t). \quad (5.23)$$

The corresponding force  $F_b^z = \delta \dot{P}_b^z$  reads

$$F_b^z = -\frac{2km}{\gamma} \left( (D-2) \gamma^2 v^2 + 1 \right) \theta(t). \quad (5.24)$$

Now consider even  $D \geq 4$ : the gamma function is regular now and we represent

$$\frac{(t^2 - r^2)^{-(D-3)/2}}{\Gamma(-\frac{D-5}{2})} = \frac{1}{\sqrt{\pi}} \left( \frac{\partial}{\partial t^2} \right)^{\frac{D-4}{2}} \frac{1}{\sqrt{t^2 - r^2}}. \quad (5.25)$$

Substituting (5.25) into (5.19) one obtains

$$\delta P_b^z = -\frac{2^{\frac{D}{2}} \pi^{\frac{D-2}{2}}}{\Gamma(\frac{D-2}{2})} \Lambda \mu \theta(t) \left( \frac{\partial}{\partial t^2} \right)^{\frac{D-4}{2}} \int_0^t \frac{r^{D-3}}{\sqrt{t^2 - r^2}} dr. \quad (5.26)$$

The variable change  $y = r^2/t^2$  leads us again to the beta function  $B(\frac{D-2}{2}, \frac{1}{2})$  so

$$\delta P_b^z = -\frac{2^{\frac{D-2}{2}} \pi^{\frac{D-1}{2}}}{\Gamma(\frac{D-1}{2})} \Lambda \mu \theta(t) \left( \frac{\partial}{\partial t^2} \right)^{\frac{D-4}{2}} t^{D-3}. \quad (5.27)$$

Applying (5.22) for integer  $\frac{D-4}{2}$ , the momentum carrying by branon reads

$$\delta P_b^z = -2(2\pi)^{\frac{D-2}{2}} \Lambda \mu \theta(t) t. \quad (5.28)$$

Substituting  $\Lambda$  (3.15), one arrives at the same expression (5.23), as for odd space-time dimensionality<sup>3</sup>. Notice that the final result keeps this form also for  $D = 2, 3$ .

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<sup>3</sup> Not surprisingly, we have the same formula. In fact, the convolution of the test function  $\varphi \in C^\infty$  with  $\delta^{(n)}(x)$ ,  $n \in \mathbb{N}$  returns its  $n$ th derivative; hence according to (5.20), the convolution with the analytic functional  $[x \theta(x)]^{-(\lambda+1)}/\Gamma(-\lambda)$  can be regarded as defining the fractional derivative of order  $\lambda$ . With this definition the differential property (5.22) becomes well defined and valid for *any*  $\lambda \in \mathbb{R}$ . The same concerns semi-integer derivatives of  $\delta(t^2 - r^2)$  in (5.21). Thus in the sense of fractional derivatives of distributions, these two ways to derive Eq. (5.23) are equivalent.

This gives rise to another problem: while all other contributions to momenta transfer are proportional to sign functions of time, (5.24) is proportional to the Heaviside function. Since  $\text{sgn}(t)$  and  $\theta(t)$  are linearly independent, this extra contribution cannot fix the above nonconservation problem. We will see shortly, that this time-asymmetric part is related to the nonzero lateral flux of the momentum.

### C. Gravitational stresses

We start by analyzing the component  $S^{z0}(h, \bar{h})$  obtained by substituting the metric deviations  $h_{MN}$  (3.3) and  $\bar{h}_{MN}$  (3.1) into (2.14). After rearrangements one obtains nonzero contributions of two types:

- the first derivatives of both  $h_{MN}$  and  $\bar{h}_{MN}$ . Using the fact that  $\bar{h}_{MN,0} = -v\bar{h}_{MN,z}$  and  $\bar{h}_{MN,00} = v^2\bar{h}_{MN,zz}$ , this contribution reduces to

$$\frac{mkv\Gamma\left(\frac{D-3}{2}\right)}{4\pi^{\frac{D-1}{2}}}\left((D-2)\gamma^2v^2+3\right)\chi_{,z}\text{sgn}(z)$$

Integrating over  $z$  by parts, one obtains  $\delta(z)$  showing that it is localized on the wall.

- Terms containing the box operator acting on  $\bar{h}_{MN}$ , namely,

$$(-2h^{00}+h^{zz})\square\bar{h}^{0z}.$$

Using the first-order Einstein equation for  $\bar{h}^{MN}$

$$\square\bar{h}^{MN} = -\kappa\left(\bar{T}^{MN} - \frac{1}{D-2}\bar{T}\eta^{MN}\right),$$

one can see that these are localized on the particle's world line.

So apparently nonlocal stresses localize on the wall and the particle world volumes. From the above calculations it is clear that it happens because one deals with the products of two Coulomb-like fields which are tight to the sources without the retardation. Thus we present the integral of  $S^{z0}$  as the sum  $\bar{S}^z$  and  $S^z$  according to their localization:

$$\int S^{z0}dzd^{D-2}\mathbf{r} = S^z + \bar{S}^z, \quad (5.29)$$

where explicitly

$$\bar{S}^z = \kappa\mu\gamma v \int (h^{zz} - 2h^{00})\delta(z-vt)dzd^{D-2}\mathbf{r}, \quad (5.30)$$

$$S^z = \frac{mkv\Gamma\left(\frac{D-3}{2}\right)}{4\pi^{\frac{D-1}{2}}}\left((D-2)\gamma^2v^2+3\right)\int\chi_{,z}\text{sgn}(z)dzd^{D-2}\mathbf{r}, \quad (5.31)$$

with  $\chi$  defined by (5.6):

$$\bar{S}^{z0} = (-2h^{00}+h^{zz})\square\bar{h}^{z0}, \quad S^{z0} = \frac{mkv\Gamma\left(\frac{D-3}{2}\right)}{4\pi^{\frac{D-1}{2}}}\left((D-2)\gamma^2v^2+3\right)\chi_{,z}\text{sgn}(z).$$

Integrating  $\bar{S}^{0z}$  over  $z$  and  $r$ , we obtain the finite contribution to the particle momentum,

$$\bar{S}^z = -2(D+1)mk\gamma v^2|t|, \quad (5.32)$$

and the corresponding time derivative is

$$\dot{\bar{S}}^z = -2(D+1)mk\gamma v^2\text{sgn}(t). \quad (5.33)$$

The second integral is evaluated by integration by parts over  $z$  and then using the arising delta function. The radial integral diverges as before:

$$S^z = -\frac{mkv\Gamma\left(\frac{D-3}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{D-2}{2}\right)}\left((D-2)\gamma^2v^2+3\right)\int r^{D-3}\chi\Big|_{z=0}dr = \quad (5.34)$$

$$= -\frac{mkv\Gamma\left(\frac{D-3}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{D-2}{2}\right)}\left((D-2)\gamma^2v^2+3\right)Q. \quad (5.35)$$

The corresponding derivative is finite:

$$f^z \equiv \delta \dot{P}_S^z = \gamma v^2 \left[ (D-2)\gamma^2 v^2 + 3 \right] mk \operatorname{sgn}(t). \quad (5.36)$$

Now consider the  $S^{00}$  component. The following contributions are nonzero:

- Terms with first derivatives of both  $h_{MN}$  and  $\bar{h}_{MN}$ :

$$\frac{\Gamma\left(\frac{D-3}{2}\right)}{4\pi^{\frac{D-1}{2}}} \left[ (D-2)\gamma^2 v^2 + 5 \right] \chi_{,z} mk \operatorname{sgn}(z).$$

These are localized to the wall integrating by parts over  $z$ .

- Terms with the second  $z$  derivatives of  $\bar{h}_{MN}$ :

$$\frac{\Gamma\left(\frac{D-1}{2}\right)}{\pi^{\frac{D-1}{2}}\gamma^2} mk |z| \chi_{,zz}.$$

These are localized to the wall integrating over  $z$  by parts twice.

- Second derivatives of  $h_{MN}$  are directly localized on the wall:

$$-\frac{\Gamma\left(\frac{D-3}{2}\right)}{\pi^{\frac{D-1}{2}}} (D-5) mk \delta(z) \chi.$$

- Boxes of  $\bar{h}_{MN}$ :

$$\bar{S}^{00} \equiv -3 h_{00} \square \bar{h}_{00} + \frac{1}{2} h_{00} \square \bar{h} + h_{PQ} \square \bar{h}^{PQ}.$$

These are localized on the particle world line after application of linearized Einstein equations.

Thus the last contribution gives the finite energy

$$\bar{S}^0 = -2 \left( (D+1)v^2 + \frac{4}{\gamma^2} \right) mk \gamma v |t|, \quad (5.37)$$

the corresponding derivative being

$$\dot{\bar{S}}^0 = -2 \left( (D+1)v^2 + \frac{4}{\gamma^2} \right) mk \gamma v \operatorname{sgn}(t). \quad (5.38)$$

The first three contributions attributed to the wall are integrated exactly as before leading to divergent total energy, but finite time derivative

$$\dot{S}^0 = mk \gamma v \left[ (D-2)\gamma^2 v^2 + 2D - 5 - \frac{2(D-3)}{\gamma^2} \right] \operatorname{sgn}(t). \quad (5.39)$$

## 6. GRAVITATIONAL DRESSING

Let us briefly summarize basic features of the particle-wall piercing collision in the perturbative approach. In zeroth order in gravitational coupling  $\varkappa$  the wall is plane, unexcited, and extending to spatial infinity. Its total momentum is constant and infinite. The particle is moving with the constant velocity orthogonally to the wall, its momentum is constant and finite. Gravitational interaction between them is repulsive and causes deceleration of the particle before the moment of perforation at  $t = 0$  and acceleration after the perforation. The perturbation of the particle world line is strictly time antisymmetric. The action of the particle gravity upon the wall is more complicated: the wall's deformation consists of the time antisymmetric component due to continuously varying gravitational force and a shock-wave component that arises after the perforation.

In the first order in  $\varkappa$  the total conserved (in Minkowskian sense) energy-momentum tensor consists of three parts: two kinetic terms and the stress tensor of the gravitational field. Since the latter is constructed from the metric deviations generated by time-independent sources, the presumably nonlocal gravitational stresses in fact localize at the unperturbed particle's world line and the wall's world volume. Therefore we can associate

the corresponding gravity contributions with kinetic terms obtaining “dressed” momenta of the particle and the wall. The associated integrated total momenta are infinite due to slow falloff of deformations at spatial infinity. To get rid of infinities we passed to time derivatives of momenta, which actually represent the total forces acting on the particle and the wall. These latter are finite and we can explore the energy-momentum balance in the form of the third Newton’s law. Note that the contribution of the shock wave makes the balance nonsymmetric in time. This contribution, however, applies only to spatial component of momentum, and does not influence the energy balance.

Now we show that one can construct the gravitationally dressed momenta of the particle and the wall such that the total momentum (4.5) satisfying the balance equation (4.11) be the sum of two but not three quantities

$$P_{\text{tot}}^M = \bar{\mathcal{P}}^M + \mathcal{P}^M. \quad (6.1)$$

For this it is enough to split the contribution of gravitational stresses between the particle and the wall according to their localization revealed in the previous section.

### A. Dressed particle momentum

Both kinetic and gravitational contributions to the particle momentum are finite, so we introduce the total dressed momentum as the sum

$$\bar{\mathcal{P}}^M = \delta\bar{P}^M + \bar{S}^M. \quad (6.2)$$

Substituting here (5.3) and (5.32) we obtain the following nonzero components:

$$\bar{\mathcal{P}}^0 = 2 [(D-3)(1-v^2) - 1] km\gamma|t|, \quad (6.3)$$

$$\bar{\mathcal{P}}^z = 2 [(D-5)v^2 + 1] km\gamma|t|. \quad (6.4)$$

Note that the gravitational stresses contribution to the energy (5.32) is negative, so the total first-order contribution to the dressed energy may have negative sign depending on the particle velocity.

### B. Dressed wall momentum

For the wall we write similarly

$$\mathcal{P}^M = \delta P^M + S^M, \quad (6.5)$$

where the kinetic contribution consists of the sum of the regular and the branon parts  $\delta P^M = \delta P_a^M + \delta P_b^M$ . Actually the branon part  $\delta P_b^M$  is nonzero only for the spatial component  $M = z$ , while for the time component we have

$$\mathcal{P}^0 = \delta P_a^0 + S^0 = \frac{\Gamma\left(\frac{D-3}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{D-2}{2}\right)} [(D-3)(1-v^2) - 1] 2mkQ(a), \quad (6.6)$$

with  $Q(a)$  given by (5.8) with  $a^2 = \gamma^2 v^2 t^2$ . This is a divergent quantity, but its time derivative is finite. Using Eq. (5.9) it is easy to establish the identity

$$\frac{d}{dt}\mathcal{P}^0 = -\frac{d}{dt}\bar{\mathcal{P}}^0, \quad (6.7)$$

showing that the change of the dressed wall’s energy per unit time is opposite to the change of the dressed particle’s energy.

For the spatial component  $\mathcal{P}^M$  we have two complications. First, the shock wave contribution  $\delta P_b^M$  is nonzero. Second, for this component the lateral flux of momentum is also nonzero. It turns out, that the regular and the branon parts as the functions of time are linearly independent, so the balance equation (4.11) must hold for them separately. So consider first the regular kinetic part  $\delta P_a^z$ . Summing up the expressions (5.15) and (5.34) one gets

$$\mathcal{P}^z = \delta P_a^z + S^z = \frac{\Gamma\left(\frac{D-3}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{D-2}{2}\right)} 2mkvQ(a). \quad (6.8)$$

Computing the time derivative of the difference between the wall and the particle momenta we find

$$\frac{d}{dt}\mathcal{P}^z = -\frac{d}{dt}\bar{\mathcal{P}}^z + km\gamma \text{sgn}(t) [(D-3)v^2 + 1]. \quad (6.9)$$

### C. The lateral flux

The origin of the extra force at the right-hand side of (6.9) lies in the nonzero flux of the  $z$  component of the brane kinetic momentum through the lateral boundary of the world tube in accordance with (4.11). After the routine consideration of all components of the energy-momentum tensor, only one contribution of the lateral flux survives, namely, the flux of the wall's  $\delta T^{zr}$  over  $dS_r = r^{D-3} \Omega_{D-3} dr dt$ . We obtain:

$$f^z \equiv \frac{d}{dt} \int T^{zr} dS_r = -\mu \Omega_{D-3} \lim_{r \rightarrow \infty} \Phi_{,r}(t, r) r^{D-3}. \quad (6.10)$$

**Antisymmetric part.** As before, we consider first the contribution of  $\Phi_a$ : substituting  $\Phi_a = -\Lambda I_a \operatorname{sgn}(t)$  and  $I_a$  from (3.16), one differentiates over  $r$  using the recurrence relations for Bessel functions,

$$\left( \frac{1}{z} \frac{\partial}{\partial z} \right) \frac{J_\nu(z)}{z^\nu} = -\frac{J_{\nu+1}(z)}{z^{\nu+1}}, \quad \left( \frac{1}{z} \frac{\partial}{\partial z} \right) \left( z^\nu J_\nu(z) \right) = z^{\nu-1} J_{\nu-1}(z), \quad (6.11)$$

and integrates over  $k$  using [19, eq. (5.11)], to get

$$f_a^z = -\frac{\mu \kappa^2 m}{4\gamma} \left( \gamma^2 v^2 + \frac{1}{D-2} \right) \left[ \operatorname{sgn}(t) - \frac{2\gamma v t}{r\sqrt{\pi}} \frac{\Gamma(\frac{D-1}{2})}{\Gamma(\frac{D-2}{2})} {}_2F_1\left(\frac{1}{2}, \frac{D-1}{2}; \frac{3}{2}; -\frac{\gamma^2 v^2 t^2}{r^2}\right) \right], \quad (6.12)$$

where the limit  $r \rightarrow \infty$  is to be taken. This results in

$$f_a^z = -km\gamma \operatorname{sgn}(t) [(D-3)v^2 + 1]. \quad (6.13)$$

This compensates for the extra terms in (6.9).

**Branon part.** As was explained above, the contributions of the branon wave to the time derivative of the wall momenta and the lateral flux must balance each other independently, as we are going to check now. The only nonzero are  $z$  components, and  $\delta P_b^z$  is defined in (5.12) with the corresponding derivative

$$F_b^z = 2\mu\Lambda \theta(t) \int I_{b,00}(t, r) d^{D-2}\mathbf{r}. \quad (6.14)$$

Substituting (3.15) and (3.17) and differentiating, one obtains

$$F_b^z = -2\mu\Lambda \Omega_{D-3} \theta(t) \int (kr)^{\frac{D-2}{2}} J_{\frac{D-4}{2}}(kr) \cos kt dr dk. \quad (6.15)$$

Integration over  $r$  leads to

$$F_b^z = -2\mu\Lambda \Omega_{D-3} \theta(t) \int k^{\frac{D-4}{2}} r^{\frac{D-2}{2}} J_{\frac{D-2}{2}}(kr) \cos kt dk \Big|_{r=\infty}. \quad (6.16)$$

On the other hand, the flux over the lateral surface  $dS_r$  is determined by the corresponding  $T^{zr}$  component of the brane's stress-energy tensor and reads

$$\int T^{zr} dS_r = -\mu \Omega_{D-3} \int \Phi_{b,r}(t, r) \delta(z) dz dt \Big|_{r=\infty} \quad (6.17)$$

while the rate of its change is given by (after the trivial  $z$  integration)

$$f_b^z = -\mu \Omega_{D-3} \Phi_{b,r}(t, r) r^{D-3} \Big|_{r=\infty}. \quad (6.18)$$

Differentiating it with the help of (6.11), one arrives at

$$f_b^z = 2\mu\Lambda \Omega_{D-3} \theta(t) \int k^{\frac{D-4}{2}} r^{\frac{D-2}{2}} J_{\frac{D-2}{2}}(kr) \cos kt dk \Big|_{r=\infty}, \quad (6.19)$$

that exactly compensates (6.16):

$$F_b^z + f_b^z = 0. \quad (6.20)$$

The computation of  $F_b^z$  in the closed form is presented in Sec. VB; hence

$$f_b^z = -\delta\dot{P}_b^z = \frac{2km}{\gamma} \left[ (D-2)\gamma^2 v^2 + 1 \right] \theta(t). \quad (6.21)$$

It is worth noting that combining two components of the lateral force, one obtains

$$\frac{d}{dt} \left[ f_a^z + f_b^z \right] = 0. \quad (6.22)$$

In other words, the total lateral  $z$  force is continuous and constant<sup>4</sup>:

$$f^z = f_a^z + f_b^z = \left[ (D-2)\gamma^2 v^2 + 1 \right] \frac{km}{\gamma}. \quad (6.23)$$

The same concerns the total  $z$  component of momentum.

## 7. CONCLUSIONS

In this paper we have analyzed the collision problem between the point particle and the domain wall in which no free momenta of colliding objects can be defined, and the energy-momentum conservation involves at any moment the contribution of the field stresses. Generically, the stresses are nonlocal, but it turns out that within the perturbation theory their contribution can be unambiguously split into two parts which are effectively localized and can be prescribed to the particle and the wall separately, leading to the notion of gravitational dressing. This is somewhat similar to introduction of the potential energy in the nonrelativistic theory, but our treatment is fully relativistic. The dressed particle momentum involves its kinetic momentum plus its “potential” momentum in the field of the wall; similarly, the wall dressed momentum involves its “potential” momentum in the field of the particle. Thus our dressing is very different from the usual dressing in the sense of adding the contribution of the proper field. We think that such a picture may be useful also in other situations in which the free states of the colliding objects cannot be defined.

The second novel feature of the particle-wall collision we have revealed here is the nonzero momentum flux through the “lateral” surface of the world tube. Because of this flux, the divergence-free stress tensor does not define the conserved energy-momentum charges as the integral over timelike sections of the world tube, since the lateral momentum flux is integrated over the time. One can still consider the change of such integrals between the infinitesimally closed surfaces, thus passing to the time derivatives of these charges. Then taking into account the lateral flux we establish the instantaneous energy-momentum balance in terms of the dressed particle and wall momenta. Actually the nonvanishing flux arises for the space component of the momentum orthogonal to the wall, while the energy is still conserved in the usual sense.

The third feature, which is also fully tractable within our model, is the excitation of the wall under the collision. Contrary to the case of colliding particles, the wall has the internal degrees of freedom that get excited in form of the branon wave. This excitation consists of two parts: one is the direct deformation of the wall in the gravitational field of the particle, which depends on their separation; another is the shock branon wave which starts after the perforation and propagates freely outward along the wall with the velocity of light. The latter gives a separate contribution to the energy-momentum which satisfies our balance equation with account for the lateral momentum flux.

Our procedure of gravitational dressing as a relativistic counterpart to the potential energy seems to be applicable to collisions of particles and branes interacting via other fields. In fact, the linearized gravity is similar to electrodynamics or any other linear field theory. The reason for “localization” of field stresses is that within the perturbational treatment of collision, the first-order field perturbations entering the field stress tensor satisfy d’Alembert equations with localized sources. This is the general features of classical relativistic collision problems.

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<sup>4</sup> We could apply the identity  $2\theta(t) - \text{sgn}(t) = 1$  directly. Doing as we do, we want to emphasize that the property is still valid at the perforation moment  $t = 0$ .

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